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High-Frequency Sum-Rule Expansion for Relativistic Quasi-One-Dimensional Quantum Plasma Dielectric Tensor IV: Radiation Effect

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A high-frequency sum-rule expansion for all transverse elements of the relativistic quasi-one-dimensional quantum plasma dielectric tensor with spin particles at $T = 0$ K in a situation where the self-consistent magnetic radiation effect is appreciable is derived. It is found that radiation energy either reduces or enhances the dispersion for finite k, depending on the direction of propagation.

1. INTRODUCTION

High-frequency sum-rule expansions of the full response tensor both for nonrelativistic and relativistic nonmagnetized and magnetized quantum plasmas without and with spins at $T = 0$ K in a situation where the self-consistent magnetic radiation effect is not ignorable are known (Genga, 1988*a*,*b*, 1992*a*, 1993*a*,*b*,*d*, 1994). However, in a situation where the self-consistent magnetic radiation effect is appreciable and hence not negligible, such as in neutron stars and pulsars, the known results pertain to those of relativistic nonmagnetized quantum plasmas (Genga, 1992*b*), nonrelativistic quasi-one-dimensional quantum plasmas (Genga, 1993*c*), and relativistic quasi-one-dimensional plasmas without spins (Genga, 1993*d*).

In this work the behavior of the full dielectric tensor in an anisotropic system of relativistic quantum plasmas with spins at $T = 0$ K up to order ω^{-5} is considered in a situation in which the self-consistent magnetic radiation effect is appreciable and therefore its contribution is not ignorable as in astrophysical plasmas. Further, as in spinless magnetized quantum plasmas (Genga, 1993*a*, *b*), the treatment is restricted to situations in

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which the order of the external magnetic field and the plasma particle density are 10^{15} G and 10^{29} particles per unit volume, respectively. At such superstrong magnetic fields, it is known (Canuto and Ventura, 1972; Genga, 1988*b*, 1992*a,b*, 1993*a–d*, 1994) that the Fermi energy of the electrons is lower than the excitation energy, i.e., $P^2/m^2 \ll \hbar\Omega$ (where Ω is the electron cyclotron frequency); only the lowest $n = 0$ level is populated, leading the mobility of the electrons to be entirely determined by the value of momentum along the z axis, i.e., p_z, resulting in a quasi-one-dimensional plasma. The high-frequency sum rules are derived by using the Hamiltonian formalism as in the above-mentioned nonrelativistic and relativistic cases. However, it is known (Goldstone, 1957; Jancovici, 1962; Genga, 1992*a,b*, 1993*b–d*, 1994) that the electron may jump from one state inside the Fermi sphere to an unoccupied state and the process creates a hole behind known as a "Fermi hole." The jump of an electron from a negative-energy state to an occupied positive-electron-energy state results in creation of a positron–electron-energy pair with a positron as a hole known as a "Dirac hole." The interaction is therefore due to both the self-consistent radiation and Coulomb. Consequently, the system is described by a set of unperturbed states which allows for positrons, photons, and electrons. Therefore, an unperturbed state must be defined by the enumerator of particles (electrons outside of the Fermi sea), the "Fermi holes," the "Dirac holes," and the photons. Jancovici (1962) therefore found it convenient to describe the interaction in the "old-fashioned" Coulomb gauge: only transverse photons exist and interact with electrons in addition to an unquantized Coulomb potential between the electrons (Goldstone, 1957). Thus, the Hamiltonian of the system is enlarged to include the photon degrees of freedom in order to allow the description by transverse interaction.

For the sake of completeness, the derivation of the polarizability tensor is reviewed below. However, in Section 2, the general relationships between the external or current–current response function sum-rule coefficients and those of the dielectric tensor are given; further, the exact ω^{-2}, ω^{-3}, ω^{-4}, ω^{-5} sum-rule coefficients are calculated. The long-wavelength limit of the results and the possible implications for the dispersion relation of high-frequency plasma modes are determined in Sections 2 and 4, respectively.

The total electron current at point x_i is given by

$$j(\mathbf{x}_i) = \frac{e}{2} \sum [\mathbf{v}_i \delta(\mathbf{x} - \mathbf{x}_i) + \delta(\mathbf{x} - \mathbf{x}_i)\mathbf{v}_i] \tag{1}$$

where $\mathbf{v}_i$ is the group velocity of the free particle i with spins. The total energy for such a particle is given by (Johnson and Lippmann, 1949;

Berestetskii *et al.*, 1978; Baym, 1974; Sakurai, 1987; Bjorken and Drell, 1964)

$$E = (\Pi^2 + m^2c^4 - 2ec\hbar \mathbf{B}^0 \cdot \mathbf{s})^{1/2} \tag{2}$$

where

$$\Pi = \mathbf{P} - \frac{e}{c}\,\mathbf{A}^0(r) = \sum_{q\sigma} \Lambda_q \varepsilon_q^\sigma (a_{q\sigma}\, e^{iq\cdot r_i} + \text{c.c.})$$

is the generalized momentum, with

$$\mathbf{A}^0(\mathbf{r}) = \frac{1}{2}\mathbf{B}^0(r) \times \mathbf{r} \tag{4}$$

the external vector potential, and

$$\Lambda_q = \frac{e}{c}\left(\frac{2\pi\hbar c^2}{V\omega_q^2}\right)^{1/2}$$

where $V = L^3$ is the volume of the system, $\omega_q = qc$, m is the rest mass of the particle, c is the speed of light in vacuum, and ε_q^σ is the unit polarization vectors satisfying the following conditions: (i) $q \cdot \varepsilon = 0$, which is obtained from the Coulomb gauge: $\nabla \cdot \mathbf{A} = 0$, and $\phi = 0$, where ϕ is the scalar potential and $\sigma = 1, 2$; and (ii) $\varepsilon_{q'}^{\sigma i} \cdot \varepsilon_q^{\sigma j} = \delta_{\sigma\sigma}\delta_{qq'}\delta_{ij}$. In addition, $a_{q\sigma}$ is the Fourier amplitude canonical to the self-consistent field vector potential, and satisfying the condition

$$a_{q\sigma} = -i\omega_q a_{q\sigma}$$

while its corresponding Hamiltonian $a_{q\sigma}^+$ satisfies the condition

$$a_{q\sigma}^+ = i\omega_q a_{q\sigma}^+$$

The group velocity of the free particle i, $(\mathbf{v}_i)$ is obtained from equation (2) as

$$\mathbf{v}_i = \gamma^{-1}\frac{\Pi_i}{m}, \tag{5}$$

where

$$\gamma = \left(1 + \frac{u^2}{2}\right)^{1/2} \tag{6}$$

is the relativistic factor with

$$\mathbf{u} = \frac{1}{m}\,[\Pi^2 - e\hbar \mathbf{B} \cdot \mathbf{s}]^{1/2} \tag{7}$$

as the nonrelativistic phase velocity of the free particle with spins. In terms of the Fourier transform, equation (1) may be expressed as

$$\langle J_{\mathbf{k}}^{\mu}(\omega)\rangle = e\langle j_{\mathbf{k}}^{\mu}(\omega)\rangle - \frac{e^2 N}{mc}\gamma^{-1} T_{\mathbf{k}}^{\mu\nu} A_{\mathbf{k}}^{0\nu}(\omega) \tag{8}$$

where

$$T_k^{\mu\nu} = \delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \tag{9}$$

Equation (8) is arrived at after taking first the Fourier transform of equation (1) followed by its expectation value; this is because the main interest is the response function of the electron system. Application of perturbation theory (Pines and Nozières, 1966; Genga, 1988*a,b*, 1992*a,b*, 1993*a–c*, 1994) gives

$$\begin{aligned}\langle j_{\mathbf{k}}^{\mu}(\omega)\rangle = {} & -\frac{e}{c}\sum_{nps} \omega^{-1}\langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle \\ & \times\left[\frac{1}{\omega - \omega_{n0}(p, p + \hbar\mathbf{k}/2) + i\eta}\right. \\ & \left. - \frac{1}{\omega - \omega_{n0}(p, p - \hbar\mathbf{k}/2 + i\eta}\right] \mathbf{A}_{\mathbf{k}}^{0\nu}(\omega)\end{aligned} \tag{10}$$

where

$$\Pi_{\mathbf{k}}^{\mu} = \frac{1}{2}\sum (\mathbf{v}_i^{\mu} e^{-i\mathbf{k}\cdot\mathbf{x}_i} + e^{-i\mathbf{k}\cdot\mathbf{x}_i}\mathbf{v}_i^{\mu}) \tag{11}$$

The arguments of ω_{n0} and the summation over $\mathbf{p}$ and $\mathbf{s}$ in equation (10) are such that $\mathbf{P} = P_z$, $\mathbf{k} = k_z$, and $\mathbf{S} = S_z$. From equations (8) and (10) combined and Ohm's law, the conductivity tensor $\sigma^{\mu\omega}(\mathbf{k}\omega)$ is obtained to be of the form

$$\sigma^{\mu\nu}(\mathbf{k}\omega) = \frac{ie^2}{\omega}\left[\chi^{\mu\nu}(\mathbf{k}\omega) + \frac{N}{m}\gamma^{-1}\mathbf{T}_{\mathbf{k}}^{\mu\nu}\right] \tag{12}$$

where $\chi^{\mu\nu}(\mathbf{k}\omega)$ is the electron response tensor with spins given by

$$\begin{aligned}\chi^{\mu\nu}(\mathbf{k}\omega) = {} & \sum_{nps}\langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle \\ & \times\left[\frac{1}{\omega - \omega_{n0}(p, p + \hbar\mathbf{k}/2) + i\eta}\right. \\ & \left. - \frac{1}{\omega - \omega_{n0}(p, p - \hbar\mathbf{k}/2) + i\eta}\right]\end{aligned} \tag{13}$$

In terms of polarizability tensor $\alpha^{\mu\nu}(\mathbf{k}\omega)$, equation (12) is expressed as

$$\begin{aligned}\alpha^{\mu\nu}(\mathbf{k}\omega) &= i\frac{4\pi e^2}{\omega^2}\sigma^{\mu\nu}(\mathbf{k}\omega)\\ &= i\frac{\omega_p^2}{\omega^2}\gamma^{-1}T_{\mathbf{k}}^{\mu\nu} + \bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)\end{aligned} \tag{14}$$

where

$$\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega) = -\frac{4\pi e^2}{\omega^2}\chi^{\mu\nu}(\mathbf{k}\omega) \tag{15}$$

The matrix elements and excitation frequencies that appear in equation (15) are those appropriate to a system of electrons with Coulomb, external magnetic field, transverse photon, and spin interactions.

2. TRANSVERSE SUM RULES

The complete modified polarizability tensor $\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ is known (Genga, 1988*a*,*b*, 1992*a*,*b*, 1993*a*–*d*, 1994) to be expressible in terms of corresponding "external" quantities $\bar{\alpha}(\mathbf{k}\omega)$ as

$$\bar{\alpha}(\mathbf{k}\omega) = \hat{\bar{\alpha}}(\mathbf{k}\omega)(\Delta - \hat{\bar{\alpha}}(\mathbf{k}\omega)^{-1}\Delta \tag{16}$$

where

$$\Delta = \mathbb{1} - n^2\mathbf{T} \tag{17}$$

with

$$\mathbf{T} = \mathbb{1} - \frac{\mathbf{k}\cdot\mathbf{k}}{k^2}$$

$$\mathbb{1} = \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Further, it is known (Genga, 1988*a*,*b*, 1992*a*,*b*, 1993*a*–*d*, 1994) that $\hat{\bar{\alpha}}(\mathbf{k}\omega)$ possess a high-frequency sum-rule expansion of the form

$$\hat{\bar{\alpha}}^{\mathrm{H}'\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{l=1\\ l=\mathrm{odd}}} \frac{\hat{\bar{\Omega}}_{l=1}^{\mu\nu}(\mathbf{k})}{\omega^{l+1}} \tag{18}$$

$$\hat{\bar{\alpha}}^{\mathrm{H}''\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{l=2\\ l=\mathrm{even}}} \frac{\hat{\bar{\Omega}}_{l=1}^{\mu\nu}(\mathbf{k})}{\omega^{l+1}} \tag{19}$$

where the superscript H denotes "Hermitian part of," with prime and double prime standing for "real part of" and "imaginary part of," respectively. The $\hat{\hat{\Omega}}^{\mu\nu}(\mathbf{k})$ coefficients are obtained from the relation

$$\hat{\hat{\Omega}}^{\mu\nu}(\mathbf{k}) = \frac{4\pi e^2}{\hbar^{l-1}} \sum_{np} \left\{ \left[\omega_{n0}\left(P, P - \frac{\hbar \mathbf{k}}{2}\right)\right]^{l-2} \langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle \right.$$
$$\left. - \left[\omega_{n0}\left(P, P - \frac{\hbar \mathbf{k}}{2}\right)\right]^{l-2} \langle 0|\Pi_{-\mathbf{k}}^{\nu}(0)|n\rangle\langle n T \Pi_{\mathbf{k}}^{\mu}(\tau)|0\rangle \right\}_{t=0} \tag{20}$$

The high-frequency expansion of $\bar{\alpha}^{\mu\nu}(\mathbf{k})$ is similarly given by equations (18) and (19) with $\bar{\Omega}_{l+1}^{\mu\nu}(\mathbf{k})$ coefficients replacing corresponding $\hat{\hat{\Omega}}_{l+1}^{\mu\nu}(\mathbf{k})$ coefficients. The relationship between the two sets of coefficients up to $l = 4$ is

$$\begin{aligned}
\bar{\Omega}_2^{\mu\nu}(\mathbf{k}) &= \hat{\hat{\Omega}}_2^{\mu\nu}(\mathbf{k}) \\
\bar{\Omega}_3^{\mu\nu}(\mathbf{k}) &= \hat{\hat{\Omega}}_3^{\mu\nu}(\mathbf{k}) \\
\bar{\Omega}_4^{\mu\nu}(\mathbf{k}) &= \hat{\hat{\Omega}}_4^{\mu\nu}(k) - \hat{\hat{\Omega}}_2^{\mu\alpha}(\mathbf{k})\hat{\hat{\Omega}}_2^{\alpha\nu}(\mathbf{k}) \\
\bar{\Omega}_5^{\mu\nu}(\mathbf{k}) &= \hat{\hat{\Omega}}_5^{\mu\nu}(k) - \hat{\hat{\Omega}}_2^{\mu\alpha}(\mathbf{k})\hat{\hat{\Omega}}_3^{\alpha\nu}(\mathbf{k}) - \hat{\hat{\Omega}}_3^{\mu\alpha}(\mathbf{k})\Omega_2^{\alpha\nu}(\mathbf{k})
\end{aligned} \tag{21}$$

The Hamiltonian of the system that satisfies equation (20) is therefore

$$H = \sum_i \gamma^{-2} \frac{\Pi_i^2}{2m} + \frac{1}{2} \sum_{\substack{ij \\ i \mp j}} U(|\mathbf{x}_i - \mathbf{x}_j|) + \sum_{q\sigma} \hbar\omega_q a_{q\sigma}^{+} a_{q\sigma} \tag{22}$$

where $U(|\mathbf{x}_i - \mathbf{x}_j|)$ is the interaction potential between a pair of particles, which is independent of velocity, and the third term on the right-hand of equation (22) is the Hamiltonian due to self-consistent magnetic radiation, with $a_{q\sigma}^{+}$ and $a_{q\sigma}$ as the "creation" and "annihilation or destruction" operators, respectively, for photons of momentum $\hbar\mathbf{q}$ and polarization σ satisfying the commutation relations (Harris, 1975; Berestetskii *et al.*, 1978; Genga, 1993*a*, *d*)

$$\begin{aligned}
[a_{q\sigma}^{\mu}, a_{q'\sigma'}^{+\mu'}] &= \delta_{qq'}\delta_{\mu\mu'}\delta_{\sigma\sigma} \\
[a_{q\sigma}^{\mu}, a_{q'\sigma'}^{\mu'}] &= [a_{q\sigma'}^{+\mu} a_{q'\sigma'}^{+\mu'}] = 0
\end{aligned} \tag{23}$$

The third term on the right-hand side of equation (22) is obtained by arguing (Berestetskii *et al.*, 1978) that $N_{q\sigma} = a_{q\sigma}^{+} a_{q\sigma}$ is very large and, hence, this results in the matrix elements of these operators also being large; thus, unity on the right-hand side of the commutation rule in equation (23) is ignorable and therefore the commutation relation in equation (23) reduces to

$$a_{q\sigma}^{+} a_{q\sigma} \simeq a_{q\sigma}^{+} a_{q\sigma} \tag{24}$$

The operations $a_{q\sigma}^{+}$ and $a_{q\sigma}$ therefore become commuting classical quantities which determine the classical field strengths in this situation. Further, Harris (1975) also argued that since the infinite zero-point energy $\hbar\omega_q/2$ of the field is an additive constant, it will cancel out when a physically meaningful quantity is calculated and therefore the Hamiltonian due to radiation becomes

$$H_{\mathrm{R}} = \sum_{q\sigma} \hbar\omega_q N_{q\sigma} \tag{25}$$

The next step is to turn to the calculation of the frequency moments (up to $l = 4$). It is known (Genga, 1988*b*, 1992*b*, 1993*a–d*, 1994) that for an anisotropic system such as the one under consideration, $\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ is nondiagonal and hence both even and odd moments of $\bar{\Omega}_{l+1}^{\mu\nu}(\mathbf{k})$ exist. The real diagonal and off-diagonal elements are also known (Genga, 1988*b*, 1992*b*, 1993*a–d*, 1994) to satisfy the symmetry condition

$$\bar{\Omega}_{l+1}^{\mu\nu}(\mathbf{k}) = \bar{\Omega}_{l+1}^{\nu\mu}(\mathbf{k}) \tag{26}$$

whereas the imaginary off-diagonal elements satisfy the antisymmetry condition

$$\bar{\Omega}_{l+1}^{\mu\nu}(\mathbf{k}) = -\bar{\Omega}_{l+1}^{\nu\mu}(\mathbf{k}) \tag{27}$$

The first moment gives rise to

$$\begin{aligned}\hat{\bar{\Omega}}_2^{\mu\nu}(\mathbf{k}) &= 4\pi e^2 \sum_{nps} \left[\frac{\langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle}{\omega_{n0}(p, p+\hbar\mathbf{k}/2)} \right. \\ &\qquad \left. + \frac{\langle 0|\Pi_{-\mathbf{k}}^{\nu}(0)|n\rangle\langle n|\Pi_{\mathbf{k}}^{\nu}(\tau)|0\rangle}{\omega_{n0}(p, p-\hbar\mathbf{k}/2)} \right]_{\tau=0} \\ &= \gamma^{-1}\omega_p^2 L_k^{\mu\nu}\end{aligned} \tag{28}$$

where

$$L_{\mathbf{k}}^{\mu\nu} = \frac{k^{\mu}k^{\nu}}{k^2} \tag{29}$$

The second moment yields

$$\begin{aligned}\hat{\bar{\Omega}}_3^{\mu\nu}(\mathbf{k}) &= \frac{4\pi e^2}{\hbar} \sum_{nps} [\langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle \\ &\qquad - \langle 0|\Pi_{-\mathbf{k}}^{\mu}(0)|n\rangle\langle n|\Pi_{\mathbf{k}}^{\mu}(\tau)|0\rangle]_{\tau=0} \\ &= \frac{2\pi e^2}{\hbar} [\langle 0|[\Pi_{\mathbf{k}}^{\mu}(\tau), \Pi_{-\mathbf{k}}^{\mu}(0)] - [\Pi_{-\mathbf{k}}^{\nu}(0), \Pi_{\mathbf{k}}^{\mu}(\tau)]|0\rangle]_{\tau=0} \\ &= i\gamma^{-2}\omega_p^2 \frac{eB^0}{mc} \eta\varepsilon^{\mu\nu\alpha}\end{aligned} \tag{30}$$

The third moment leads to

$$\hat{\hat{\Omega}}_4^{\mu\nu}(\mathbf{k}) = \frac{4\pi e^2}{\hbar^2} \sum_{nps} \left[\omega_{n0}\left(p, p - \frac{\hbar \mathbf{k}}{2}\right) \langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle \right.$$

$$\left. + \omega_{n0}\left(p, p + \frac{\hbar \mathbf{k}}{2}\right) \langle 0|\Pi_{-\mathbf{k}}^{\nu}(0)|n\rangle\langle n|\Pi_{\mathbf{k}}^{\mu}(\tau)|0\rangle \right]_{\tau=0}$$

$$= \frac{2\pi e^2}{\hbar^2} \langle 0|[[\Pi_{\mathbf{k}}^{\mu}(\tau), H], \Pi_{-k}^{\nu}(0)] + [[\Pi_{-\mathbf{k}}^{\mu}(0), H], \Pi_{\mathbf{k}}^{\mu}(\tau)]|0\rangle|_{\tau=0}$$

$$= \gamma^{-4} \frac{\omega_p^2 e B_\eta^0}{2m^2 c} k^\alpha k^\mu \langle 0|2mc(eB_\eta^0)^{-1} \frac{\partial^2}{\partial x^\alpha \partial x^\nu} + i\varepsilon^{\alpha\eta\beta} x^\beta \frac{\partial}{\partial x^\nu} - i\varepsilon^{\alpha\eta\nu} x^\alpha \frac{\partial}{\partial x^\alpha}$$

$$- \varepsilon^{\mu\nu\alpha} \frac{eB_\eta^0}{2mc} (x^\mu)^2 |0\rangle - \gamma^{-4} \frac{\omega_p^2 e B_\eta}{2m^2 c} k^\alpha k^\nu \langle 0|2mc(eB_\eta^0)^{-1} \frac{\partial^2}{\partial x^\alpha \partial x^\mu}$$

$$+ i\varepsilon^{\alpha\eta\beta} x^\beta \frac{\partial}{\partial x^\mu} - i\varepsilon^{\alpha\eta\mu} x^\alpha \frac{\partial}{\partial x^\alpha} - \varepsilon^{\mu\eta\alpha} \frac{eB_\eta^0}{2mc} (x^\mu)^2 |0\rangle$$

$$- \gamma^{-4} \omega_p^2 e B_\eta^0 k^\alpha k^\alpha \langle 0|2mc(eB_\eta^0)^{-1} \frac{\partial^2}{\partial x^\mu \partial x^\nu} - i\varepsilon^{\alpha\eta\nu} x^\alpha \frac{\partial}{\partial x^\mu}$$

$$- i\varepsilon^{\alpha\eta\mu} x^\alpha \frac{\partial}{\partial x^\nu} + [\varepsilon^{\mu\eta\alpha}(x^\alpha)^2 \delta^{\mu\nu} - \varepsilon^{\mu\eta\nu} x^\mu x^\nu] \frac{eB_\eta^0}{2mc} |0\rangle$$

$$+ \gamma^{-4} \omega_p^4 \frac{\hbar k^2}{mN} \sum_{q\sigma} \frac{N_{q\sigma}}{\omega_q} \delta^{\mu\nu} + \gamma^{-2} \omega_p^4 \langle 0|L_{\mathbf{k}}^{\mu\nu} + \frac{1}{N} \sum_q L_q^{\mu\nu}(S_{\mathbf{k}-q} - S_{\mathbf{k}})|0\rangle \tag{31}$$

The fourth moment is given by

$$\hat{\hat{\Omega}}_5^{\mu\nu}(\mathbf{k}) = \frac{4\pi e^2}{\hbar^3} \sum_{nps} \left\{ \left[\omega_{n0}\left(p, p - \frac{\hbar k}{2}\right) \right]^2 \langle 0|\Pi_{\mathbf{k}}^{\mu}(\tau)|n\rangle\langle n|\Pi_{-\mathbf{k}}^{\nu}(0)|0\rangle \right.$$

$$\left. - \left[-\omega_{n0}\left(p, p + \frac{\hbar k}{2}\right) \right]^2 \langle 0|\Pi_{\mathbf{k}}^{\nu}(0)|n\rangle\langle n|\Pi_{\mathbf{k}}^{\mu}(\tau)|0\rangle \right\}_{\tau=0}$$

$$= \frac{2\pi e^2}{\hbar^3} \langle 0|[[[\Pi_k^{\mu}(\tau), H], H], \Pi_{-\mathbf{k}}^{\nu}(0)]$$

$$- [[[\Pi_{-\mathbf{k}}^{\nu}(0), H], H], \Pi_{\mathbf{k}}^{\mu}(\tau)]|0\rangle|_{\tau=0}$$

$$= -\gamma^{-6} \frac{\omega_p^2 e B_\eta^0}{4m^2 c} k^\alpha k^\mu \langle 0|i \frac{7}{4} \varepsilon^{\mu\alpha\nu} \frac{\partial^2}{\partial x^\alpha \partial x^\alpha} + \frac{17}{8} \varepsilon^{\nu\eta\alpha} \frac{eB_\eta^0}{mc} x^\nu \frac{\partial}{\partial x^\alpha}$$

$$+ i \frac{7}{4} \varepsilon^{\nu\eta\alpha} \frac{(eB_\eta^0)^2}{m^2 c^2} (x^\nu)^2 |0\rangle - \gamma^{-6} \frac{\omega_p^2 e B_\eta^0}{4m^2 c} k^\alpha k^\nu \langle 0|i \frac{7}{4} \varepsilon^{\mu\alpha\eta} \frac{\partial^2}{\partial x^\alpha \partial x^\alpha}$$

$$+\frac{17}{8}\varepsilon^{\mu\alpha\eta}\frac{eB_\eta^0}{mc}x^\mu\frac{\partial}{\partial x^\alpha}+i\frac{7}{4}\varepsilon^{\mu\alpha\eta}\frac{(eB_\eta^0)^2}{m^2c^2}(x^\mu)^2|0\rangle$$

$$-\gamma^{-6}\frac{\omega_p^2 eB_\eta^0}{4m^2c}k^\alpha k^\alpha\langle 0|i6\varepsilon^{\mu\eta\alpha}\frac{\partial^2}{\partial x^\alpha\partial x^\nu}+i6\varepsilon^{\nu\alpha\eta}\frac{\partial^2}{\partial x^\alpha\partial x^\mu}$$

$$+i\frac{3}{2}\varepsilon^{\mu\nu\nu}\frac{\partial^2}{\partial x^\alpha\partial x^\alpha}+\frac{3}{2}\varepsilon^{\mu\eta\alpha}\delta^{\mu\nu}\frac{eB_\eta^0}{mc}+\varepsilon^{\nu\eta\alpha}\frac{eB_\eta^0}{mc}x^\nu\frac{\partial}{\partial x^\nu}$$

$$+3\varepsilon^{\nu\eta\alpha}\frac{eB_\eta^0}{mc}x^\nu\frac{\partial}{\partial x^\mu}-i\frac{15}{4}\varepsilon^{\mu\eta\nu}\frac{(eB_\eta^0)^2}{m^2c^2}(x^\mu)^2+i\varepsilon^{\mu\eta\nu}(x^\nu-x^\mu)\frac{eB_\eta^0}{mc}|0\rangle$$

$$-i\gamma^{-6}\frac{\omega_p^4 e\hbar}{2Nm^2c}\sum_q\frac{N_{q\sigma}}{\omega_q}[4k^\alpha k^\alpha\varepsilon^{\mu\nu\eta}B_\eta^0+11(k^\alpha k^\mu\varepsilon^{\alpha\eta\nu}+k^\alpha k^\nu\varepsilon^{\nu\eta\nu})B_\eta^0]$$

$$+i\gamma^{-4}\omega_p^2\frac{eB_\eta^0}{2mc}\langle 0|L_k^{\mu\nu}+\frac{1}{N}\sum_q(\varepsilon^{\mu\eta\alpha}L_q^{\alpha\nu}+\varepsilon^{\alpha\eta\nu}L_q^{\alpha\mu})(S_{\mathbf{k}-\mathbf{q}}-S_{\mathbf{k}})|0\rangle \tag{32}$$

An explicit expression for $\hat{\hat{\Omega}}_{l+1}^{\mu\nu}(k)$, is obtained by choosing a k system (Genga, 1988*a*,*b*, 1992*a*,*b*, 1993*a*–*d*, 1994) in which

$$\mathbf{k}=(0,0,k) \tag{33}$$

$$\mathbf{B}^0=(B^0\sin\theta, 0, B^0, \cos\theta) \tag{34}$$

and

$$q=(q\sin\theta\cos\theta, q\sin\theta\sin\phi, q\cos\theta) \tag{35}$$

Landau gauge

$$\mathbf{A}^0=\frac{1}{2}(0, B_z^0x-zB_x^0, 0) \tag{36}$$

is applied to obtain equation (34).

3. LONG-WAVELENGTH LIMIT

In the long-wavelength ($k\to 0$) limit equations (32)–(35) become

$$\hat{\hat{\Omega}}_2^{11}(\mathbf{k})=\hat{\hat{\Omega}}_2^{22}(\mathbf{k})=0$$

$$\hat{\hat{\Omega}}_2^{33}(\mathbf{k})=\gamma^{-1}\omega_p^2$$

$$\hat{\hat{\Omega}}_3^{12}(\mathbf{k})=-\hat{\hat{\Omega}}_3^{21}(\mathbf{k})=i\gamma^{-2}\omega_p^2\Omega\cos\theta$$

$$\hat{\hat{\Omega}}_3^{23}(\mathbf{k})=-\hat{\hat{\Omega}}_3^{32}(\mathbf{k})=i\gamma^{-2}\omega_p^2\Omega\sin\theta$$

$$\hat{\hat{\Omega}}_4^{13}(\mathbf{k})=\hat{\hat{\Omega}}_4^{31}(\mathbf{k})=0$$

$$\hat{\hat{\Omega}}_4^{11}(\mathbf{k}) = \hat{\hat{\Omega}}_4^{22}(\mathbf{k}) = -\gamma^{-2}\frac{\omega_p^2}{m}\left(\frac{2}{15}E_{\mathrm{corr}} - \gamma^{-2}E_{\mathrm{R}}\right)k^2$$

$$\hat{\hat{\Omega}}_4^{33}(\mathbf{k}) = \gamma^{-2}\omega_p^4 - \gamma^{-4}\frac{\omega_p^2}{m}\left(6E_{\mathrm{F}} - \frac{4}{15}\gamma^2 E_{\mathrm{corr}} - E_{\mathrm{R}}\right)k^2$$

$$\hat{\hat{\Omega}}_5^{12}(\mathbf{k}) = -\hat{\hat{\Omega}}_5^{21}(\mathbf{k}) = -i\gamma^{-6}\frac{\omega_p^2\Omega}{8m}\left(6E_{\mathrm{F}} + \frac{16}{15}\gamma^2 E_{\mathrm{corr}} + 16E_{\mathrm{R}}\right)k^2\cos\theta$$

$$\hat{\hat{\Omega}}_5^{23}(\mathbf{k}) = -\hat{\hat{\Omega}}_5^{32}(\mathbf{k}) = -i\gamma^{-6}\frac{\omega_p^2\Omega}{8m}\left(30E_{\mathrm{F}} - \frac{24}{15}\gamma^2 E_{\mathrm{corr}} + 44E_{\mathrm{R}}\right)k^2\sin\theta \tag{37}$$

where $E_{\mathrm{F}} = (P_{\mathrm{F}}^{(0)})^2/2m$ is the lowest Landau level Fermi energy per particle, $E_{\mathrm{corr}} = N\sum_q (4\pi e^2/q^2)g_q$ is the correlation energy (which is negative), $E_{\mathrm{R}} = (\hbar/mN)\sum_{q\sigma}(4\pi e^2/\omega_q)N_{q\sigma}$ is the self-consistent magnetic radiation energy, and $\Omega = eB^0/mc$ is the electron cyclotron frequency.

The eigenstate $|0\rangle$ is expressed as (Johnson and Lippmann, 1949)

$$|0\rangle = (2\pi)^{-1/2}\lambda^{-1}\exp\left[\frac{(y-y_0)^2}{4\lambda^2} + \frac{iP_z}{\hbar}\right] \tag{38}$$

with

$$\lambda = -\hbar/m\Omega$$
$$y_0 = -2\frac{c}{e}P_x \tag{40}$$
$$y = B_z^0 x - zB_x^0 = -\frac{c}{e}P_y$$

4. RADIATION EFFECT

The radiation effect on the undamped high-frequency, quasi-one-dimensional quantum plasma dispersion with spins at $T = 0$ K is determined by using the high-frequency sum rules (HFSR-s). The high-frequency modes under consideration are the "ordinary" mode and the "extraordinary" mode with cutoff frequency $\omega_2 = [1 + (1 + 4\omega_p^2/\Omega^2)^{1/2}]$ propagating both along and across the external magnetic field, respectively.

4.1. Propagation Parallel to Magnetic Field

In this case, it is known (Genga, 1988*a,b*, 1992*a,b*, 1993*a–d*, 1994) that only the longitudinal and extraordinary modes with respective dispersion relations

$$\varepsilon_{33}(\mathbf{k}\omega) = 1 + \alpha_{33}(\mathbf{k}\omega) = 0 \tag{41}$$

and

$$[\omega_{11}(\mathbf{k}\omega) = n^2(\mathbf{k}\omega)]^2 - \omega_{12}(\mathbf{k}\omega) = 0 \tag{42}$$

exist. Application of a small perturbation to equations (41) and (42) leads to the ensuing plasmon frequency of oscillation of the form

$$\omega^2(\mathbf{k}) = \gamma^{-1}\omega^2\left[1 - \gamma^{-5/2}\frac{\omega_p^2}{m}\left(6E_\mathrm{F} - \frac{4}{15}\gamma^2 E_\mathrm{corr} - E_\mathrm{R}\right)k^2\right] \tag{43}$$

for the longitudinal plasmon and

$$\omega^2 = \gamma^{-1}\omega_2^2\left\{1 + \frac{2}{3}\gamma^{1/2}\left[\frac{c^2}{\omega_p^2} + \gamma^{-2}\frac{\omega_p^2}{m\omega_2^4}\left(\frac{2}{15}E_\mathrm{corr} - \gamma^2 E_\mathrm{R}\right)\right]k^2\right\} \tag{44}$$

for the extraordinary mode, respectively. Equation (43) shows that radiation energy reduces the negative dispersion for finite k. In equation (44) it can be seen that it also reduces the positive refractive dispersion for finite k.

4.2. Propagation Perpendicular to Magnetic Field

In this case a pure transverse mode called the "ordinary mode" and a coupled transverse–longitudinal mode exist which are determined by dispersion relations

$$\varepsilon_{11}(\mathbf{k}\omega) - n^2(\mathbf{k}\omega) = 0 \tag{45}$$

and

$$[\varepsilon_{22}(\mathbf{k}\omega) - n^2(\mathbf{k}\omega)]\varepsilon_{33}(\mathbf{k}\omega) - \varepsilon_{23}^2(\mathbf{k}\omega) = 0 \tag{46}$$

respectively. When a small perturbation is applied to equation (45) it is found that there is no shift frequency, thus leading to the conclusion that there is no ordinary mode, contrary to expectation. However, for equation (6) the frequency of oscillations is of the form

$$\omega^2(\mathbf{k}) = \gamma^{-1}\omega_2^2\left[1 + \gamma^{1/2}\left[\frac{c^2}{\omega_p^2} - \frac{2\gamma^{-4}}{m\omega_2^2}\left(3E_\mathrm{F} - \frac{1}{15}\gamma^2 E_\mathrm{corr} - E_\mathrm{R}\right)k^2\right]\right] \tag{47}$$

Equation (47) shows that the radiation energy enhances the positive refractive dispersion for finite k.

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